Columbia International Publishing
Contemporary Mathematics and Statistics
(2014) Vol. 2 No. 1 pp. 55-75
doi:10.7726/cms.2014.1004
Research Article

Stochastic Processes from $\mathbf{F}_{\psi}(\Omega)$ Spaces

Yuriy Kozachenko[1] and Yuriy Mlavets[2*]

Received 14 August 2014; Published online 13 September 2014

Abstract

In the paper we investigated the basic properties of the spaces $F_{\psi}(\Omega)$. There were also obtained the estimates for the distribution of suprema on a compact set for the stochastic process from such spaces. The probabilities of large deviations for the sums of independent stochastic processes from the space $F_{\psi}(\Omega)$ have been considered and the estimates for the distribution of suprema on R for the stochastic processes from such spaces have been found.

Keywords: Banach space; $F_{\psi}(\Omega)$ -space; Majorizing characteristic; Condition $\mathbf{H}$; Stochastic process; Probability of large deviations; Compact set; Metric massiveness

1. Introduction

The very first result devoted to the investigation of the local properties of stochastic processes belongs to A.M. Kolmogorov. His theorem on sample continuity with probability one was published in the paper by Slutsky (1937). This theorem has created a trend in the theory of stochastic processes. The other fondamental work in this approach devoted to the general conditions of the sample continuity and belonging to the Lipshitz class of random fields, is the monogrhaph by Yandrenko (1980). The paper by Kozachenko and Yadrenko (1976) includes the similar conditions for different clases of random fields. The conditions for continuity of random functions defined on compact set in the Hilbert space were studied in the work by Skorokhod (1973).

Many scientists investigated the properties of the distributions for suprema of stochastic processes and the problems of existence for the ordinary and exponential moments of the distribution for the supremum of a process. Much attantion has been paid to the problem of finding the estimates for

*Corresponding e-mail: yura-mlavec@ukr.net
1 Department of Probability Theory, Statistics and Actuarial Mathematics, Taras Shevchenko National University of Kyiv, Kyiv 01601, Ukraine
2* Department of Cybernetics and Applied Mathematics, Uzhhorod National University, Uzhhorod 88000, Ukraine

the probability $P\left\{\sup_{t\in T}|X(t)|>\varepsilon\right\}$. The results connected to these questions were published in the books by Cramer & Leadbetter (1967), Marcus & Pisier (1981), Ledoux & Talagrand (1991), Buldygin & Kozachenko (1998). Such scientists as Albin (1998), Ledoux (1990), Ostrovsky (1990) were involved into the investigation of properties of the stochastic processes from some particular classes.

In the 60s of the XX century the local properties of the Gaussian processes had been studied. In particular, Belyaev (1961) had gotten the conditions of continuity for the stationary Gaussian processes in the terms of spectral functions and the well-known "Belyaev alternative". Using the different approaches, the conditions of continuity for the Gaussian processes had been obtained by Dudley (1965) and Delporte (1964). The fundamental results regarding the properties of the Gaussian stochastic processes were made by Lindgren (1971), Dudley (1973), Borell (1978), Talagrand (1987), Piterbarg (1996). The paper by Bondarenko & Ivanov (1992) is devoted to the investigation of the properties of the sample paths for the random fields with stable increments.
Skorokhod (1970), Landau & Shepp (1970), Ledoux & Talagrand (1991), Lifshits (1995) focused in their investigations on estimation of the exponential moments and the distribution of the supremum for the Gaussian processes.

In the 1960s there appeared a very important work considering the wider class of random variables and processes than Gaussian. Namely, the notion of the sub-Gaussian random variable was introduced by Kahane (1960). Later on, it was proved by Buldygin and Kozachenko (1980) that the space of sub-Gaussian random variables is a Banach space with regard to sub-Gaussian standard. The properties and different applications of the sub-Gaussian and strictly sub-Gaussian random variables were investigated in the papers by Buldygin & Kozachenko (1998), Giuliani et al. (2002), Ostrovsky (1990), Pashko (1998).

Moreover, Kozachenko (1968) indroduced the notion of sub-Gaussian stochastic processes. The properties of such class of stochastic processes were studied in the works by Buldygin (1977) , Jain & Marcus (1978). The exponential moments and the estimates for distribution of suprema for sub-Gaussian and related processes were considered in the works by Kahane (1960), Ostrovsky (1990), Fukuda (1990), Ledoux & Talagrand (1991).

Kozachenko & Ostrovsky in 1985 introduced the notion of random variables and processes of sub-Gaussian type, namely, $Sub_{\varphi}(\Omega)$, which generalizes the spaces of sub-Gaussian random variables. The spaces of φ-sub-Gaussian random variables are spaces of centered random variables with particular growth of the exponential moments. Properties of these spaces, estimates and convergence conditions for sums of independent random variables from these spaces were investigated in the monograph by Buldygin & Kozachenko (1998). The properties of the φ-sub-Gaussian spaces were also studied in the work by Giuliani et al. (2003) and in the monograph by Vasylyk et al. (2008). The estimates for the distribution of the suprema for φ- sub-Gaussian stochastic processes were investigated in the work by Kozachenko et al. (2003).

Pre-Gaussian stochastic processes were introduced in the work by Buldygin & Kozachenko (1974). Their properties and the distribution of suprema were studied in the works by Buldygin & Kozachenko (1993), Dmytrovsky (1981).

The space $F_\psi(\Omega)$ had been introduced in the paper by Yermakov & Ostrovsky (1986). It was proved there that it is a Banach space with regard to the norm $\|\xi\|_\psi = \sup_{u\ge1}\frac{\left(\mathrm{E}|\xi|^u\right)^{1/u}}{\psi(u)}$. In the paper by Kozachenko and Mlavets (2012) the basic properties of the space $F_\psi(\Omega)$ were studied, the connection to Orlicz spaces was established and the estimates for the distribution of the suprema of the stochastic processes from these spaces were found.

This paper consists of introduction and four sections. In the second section the main properties of the spaces $F_\psi(\Omega)$ are investigated. The section 3 includes the estimates for the distribution of suprema of stochastic processes defined on compact set and belonging to the space $F_\psi(\Omega)$. In the forth section we consider the probabilities of large deviations for the sums of independent stochastic processes from the space $F_\psi(\Omega)$. The estimates for the distribution of suprema on R for the stochastic processes from the space $F_\psi(\Omega)$ are founded in the fifth section.

2. $F_\psi(\Omega)$ - spaces

***Definition* 2.1** (Kozachenko and Mlavets, 2012) Let $\psi(u)>0$, $u\ge1$ be a monotonicaly increasing continuous function for which $\psi(u)\to\infty$ as $u\to\infty$. A random variable ξ belongs to the space $F_\psi(\Omega)$ if the following is true:

$$\sup_{u\ge1}\frac{\left(E|\xi|^u\right)^{1/u}}{\psi(u)}<\infty.$$

The space $F_\psi(\Omega)$ is a Banach space equipped with a norm

$$\|\xi\|_\psi=\sup_{u\ge1}\frac{\left(E|\xi|^u\right)^{1/u}}{\psi(u)}.$$

***Theorem* 2.1** (Kozachenko and Mlavets, 2012) *If a random variable ξ belongs to the space $F_\psi(\Omega)$, then for any $\varepsilon>0$ the following inequality holds true:*

$$P\{|\xi|>\varepsilon\}\le\inf_{u\ge1}\frac{\|\xi\|_\psi^u(\psi(u))^u}{\varepsilon^u}.$$

***Theorem* 2.2** (Kozachenko and Mlavets, 2012) *If a random variable ξ belongs to the space $F_\psi(\Omega)$ and $\psi(u)=u^\alpha$, where $\alpha>0$, then for any $\varepsilon\ge e^\alpha\|\xi\|_\psi$ the inequality holds:*

$$P\{|\xi| > \varepsilon\} \le \exp\left\{ -\frac{\alpha}{e}\left(\frac{\varepsilon}{\|\xi\|_\psi}\right)^{1/\alpha} \right\}.$$

***Theorem* 2.3** (Kozachenko and Mlavets, 2012) *If a random variable ξ belongs to the space $F_\psi(\Omega)$ and $\psi(u) = e^{au^\beta}$, where $a > 0$, $\beta > 0$, then for any $\varepsilon \ge e^{a(\beta+1)}\|\xi\|_\psi$ the following is true:*

$$P\{|\xi| > \varepsilon\} \le \exp\left\{ -\frac{\beta}{a^{1/\beta}}\left(\frac{\ln \frac{\varepsilon}{\|\xi\|_\psi}}{\beta+1}\right)^{\frac{\beta+1}{\beta}} \right\}.$$

***Definition* 2.2** (Kozachenko and Mlavets, 2012) A positive nondecreasing number sequence $(\kappa(n), n \ge 1)$ is said to be M -characteristic (majorizing characteristic) for the space $F_\psi(\Omega)$, if for any random variables ξ_i, $i = 1,2,\ldots,n$ from this space the following inequality holds:

$$\left\| \max_{1\le i\le n} |\xi_i| \right\|_\psi \le \kappa(n) \max_{1\le i\le n} \|\xi_i\|_\psi .$$

***Theorem* 2.4** (Kozachenko and Mlavets, 2012) *The sequence*

$$\kappa(n) = \sup_{u\ge 1} \inf_{v>0} n^{\frac{1}{u+v}} \frac{\psi(u+v)}{\psi(u)}$$

is the majorizing characteristics for the space $F_\psi(\Omega)$.

***Theorem* 2.5** (Kozachenko and Mlavets, 2012) *The sequence*

$$\kappa(n) = \frac{1}{e^a} \exp\left\{ S(a,\beta)(\ln n)^{\frac{\beta}{\beta+1}} \right\},$$

where $S(a,\beta) = (\beta a)^{\frac{1}{\beta+1}}(\beta^{-1}+1)$ is the majorizing characteristics for the space $F_\psi(\Omega)$, where $\psi(u) = e^{au^\beta}$, $a > 0$, $\beta > 0$, and $\kappa(1) = 1$.

***Definition* 2.3** (Kozachenko and Mlavets, 2012) We shall say that the condition $\mathbf{H}$ for the Banach spaces $B(\Omega)$ of random variables is fulfilled if there exists such an absolute constant C_B that for any centered and independent random variables $\xi_1, \xi_2, \ldots, \xi_n$ from $B(\Omega)$ the following is true:

$$\left\| \sum_{i=1}^{n} \xi_i \right\|^2 \le C_B \sum_{i=1}^{n} \|\xi_i\|^2 .$$

The constant C_B is called a scale constant for the space $B(\Omega)$. For all spaces $F_\psi(\Omega)$ we shall denote the constants $C_{F_\psi(\Omega)}$ as C_ψ.

The general conditions when the condition $\mathbf{H}$ is fulfilled for the space $F_\psi(\Omega)$ are found in the paper by Kozachenko and Mlavets (2012). In particular, there was shown that the condition $\mathbf{H}$ is fulfilled for the space $F_\psi(\Omega)$ if $\psi(u) = u^\alpha$, $\alpha \geq \frac{1}{2}$. It's worse to mention that if $\alpha < \frac{1}{2}$ then the condition $\mathbf{H}$ for this space is not fulfilled.

3. Estimates for the distribution of suprema on compact set for the stochastic processes from $F_\psi(\Omega)$ spaces

***Definition* 13.1** It is said that a stochastic process $X = \{X(t), t \in T\}$, where T is some set, belongs to the space $F_\psi(\Omega)$ if for any $t \in T$ the random variable $X(t)$ belongs to the space $F_\psi(\Omega)$.

***Definition* 3.22** As metric massiveness $N(u)$ of the compact metric space (T, ρ) we shall call the least number of close circles with radius less or equal to u and covering the set T.

***Theorem* 3.13** (Kozachenko and Mlavets, 2012) *Let $T = (T, \rho)$ be a compact metric space, $N(u)$ be a metric massiveness of the space (T, ρ), $X = \{X(t), t \in T\}$ be a separable stochastic process from the space $F_\psi(\Omega)$, $\kappa(n)$ be the majorizing characteristics of the space $F_\psi(\Omega)$ and $\kappa(u)$, $u \geq 1$ be any monotonically increasing function coinciding with $\kappa(n)$ for natural numbers $n \geq 1$. Assume there exists such a function*

$$\sigma = \left\{ \sigma(h), 0 \leq h \leq \sup_{t,s \in T} \rho(t,s) \right\},$$

that $\sigma(h)$ is continuous, monotonically increasing, $\sigma(0) = 0$ and

$$\sup_{\rho(t,s) \leq h} \|X(t) - X(s)\| \leq \sigma(h).$$

If for any $z > 0$ the following condition is fulfilled

$$\int_0^z \kappa\left(N\left(\sigma^{(-1)}(u)\right)\right) du < \infty,$$

where $\sigma^{(-1)}(u)$ is an inverse function for $\sigma(u)$, then the random variable $\sup_{t \in T} |X(t)|$ belongs to the space $F_\psi(\Omega)$ with probability one and

$$\left\| \sup_{t\in T} |X(t)| \right\| \le B(p),$$

where $B(p) = \inf_{t\in T} \|X(t)\| + \frac{1}{p(1-p)} \int_0^{\gamma p} \kappa\left(N\left(\sigma^{(-1)}(u)\right)\right) du$, $\gamma = \sigma\left(\sup_{t,s\in T} \rho(t,s)\right)$,

a number p *satisfies* $0 < p < 1$.

***Corollary* 3.14** (Kozachenko and Mlavets, 2012) *Let a process* $X = \{X(t), t \in T\}$ *belonging to the space* $F_\psi(\Omega)$, *satisfy the conditons of the theorem 3.1, then the following inequality holds true for any* $\varepsilon > 0$:

$$P\left\{\sup_{t\in T} |X(t)| > \varepsilon\right\} \le \inf_{u\ge 1} \frac{B^u(p)(\psi(u))^u}{\varepsilon^u}.$$

***Corollary* 3.25** (Kozachenko and Mlavets, 2012) *Let* $X = \{X(t), t \in [c,d]\}$, $-\infty < c < d < +\infty$ *be a separable stochastic process from the space* $F_\psi(\Omega)$. *Assume, the following condition is fulfilled for it*

$$\sup_{\substack{|t-s|\le h \\ t,s\in[c,d]}} \|X(t) - X(s)\|_\psi \le \sigma(h),$$

where $\sigma = \{\sigma(h), 0 \le h \le d - c\}$ *is a continuous monotonically increasing function and* $\sigma(0) = 0$. *If for any* $z > 0$ *the additional condition is true*

$$\int_0^z \kappa\left(\frac{d-c}{2\sigma^{(-1)}(u)} + 1\right) du < \infty,$$

then $\sup_{t\in[c,d]} |X(t)| \in F_\psi(\Omega)$ *with probability one and for any* $0 < p < 1$ *the following inequality holds true*

$$\left\| \sup_{t\in[c,d]} |X(t)| \right\|_\psi \le \tilde{B}(p),$$

where $\widetilde{B}(p) = \inf_{t\in[c,d]} \|X(t)\|_\psi + \frac{1}{p(1-p)} \int_0^{\gamma p} \kappa\left(\frac{d-c}{2\sigma^{(-1)}(u)} + 1\right) du$, $\gamma = \sigma(d-c)$, $\kappa(u)$ *is the mojorizing characteristics for the space* $F_\psi(\Omega)$, $\sigma^{(-1)}(u)$ *is an inverse function for* $\sigma(u)$.

Furthermore, for any $\varepsilon > 0$ *the following inequality holds true:*

$$P\left\{\sup_{t\in[c,d]} |X(t)| > \varepsilon\right\} \le \inf_{u\ge 1} \frac{\widetilde{B}^u(p)(\psi(u))^u}{\varepsilon^u}.$$

***Corollary* 3.36** (Kozachenko and Mlavets, 2012) *Let* $X = \{X(t), t \in [c,d]\}$, $-\infty < c < d < +\infty$ *be a separable stochastic process from the space* $F_\psi(\Omega)$ *and for some* $0 < \mu < 1$ *the following condition is true*

$$\sup_{\substack{|t-s|\le h \\ t,s\in[c,d]}} \|X(t) - X(s)\|_\psi \le \frac{C}{\left(\kappa\left(\frac{d-c}{2h} + 1\right)\right)^{1/\mu}},$$

where $C > 0$ *is some constant,* $h < d - c$. *Then,* $\sup_{t\in[c,d]} |X(t)|$ *belongs to the space* $F_\psi(\Omega)$ *with probability one and the following is true*

$$\left\|\sup_{t\in[c,d]} |X(t)|\right\|_\psi \le \inf_{t\in[c,d]} \|X(t)\|_\psi + C\left(\kappa\left(\frac{3}{2}\right)\right)^{(\mu-1)/\mu} \frac{(1+\mu)^{\mu+1}}{\mu^\mu(1-\mu)} = \widetilde{B}.$$

Moreover, for any $\varepsilon > 0$ *it is true that*

$$P\left\{\sup_{t\in[c,d]} |X(t)| > \varepsilon\right\} \le \inf_{u\ge 1} \frac{\widetilde{B}^u(\psi(u))^u}{\varepsilon^u}.$$

4. Probabilities of large deviations for sum of independent stochastic processes from the spaces $F_\psi(\Omega)$

***Lemma* 4.17** *Let* $\xi \in F_\psi(\Omega)$, $p \ge 1$. *Then*

$$\|E|\xi|\|_\psi \le \|\xi\|_\psi.$$

Proof. If m is an arbitrary constant then

$$\|m\|_{\psi} = \sup_{u\geq 1}\frac{\left(E|m|^{u}\right)^{1/u}}{\psi(u)} = \sup_{u\geq 1}\frac{m}{\psi(u)} = \frac{m}{\psi(1)}. \quad (1)$$

It follows from the definition of the norm $\|\xi\|_{\psi}$ that

$$E|\xi| \leq \|\xi\|_{\psi}\psi(1). \quad (2)$$

So, the proof of the lemma are completed acording to the equality (1) and inequality (2).

***Theorem* 4.18** *Let (T,ρ) be a compact metric space, $Y=\{Y(t),\ t\in T\}$ be a stochastic process belonging to the space $F_{\psi}(\Omega)$ and satisfying condition $\mathbf{H}$ with constant C_{ψ}. We assume that Y is the separable process on (T,ρ). Moreover, it is assumed that there exists such a continuous monotonically increasing function $\sigma(h)$ $(\sigma(0)=0)$ that*

$$\sup_{\rho(t,s)\leq h}\|Y(t)-Y(s)\|_{\psi} \leq \sigma(h)$$

and for any $z>0$ the following condition is fulfilled

$$\int_0^z \kappa\left(N\left(\sigma^{(-1)}(u)\right)\right)du < \infty,$$

where $\kappa(n)$ is a majorizing characteristics of the space $F_{\psi}(\Omega)$.

Let $X(t)=Y(t)-m(t)$, where $m(t)=EX(t)$ and $X_k(t)$ are independent copies of the process $X(t)$, $S_n(t)=\frac{1}{\sqrt{n}}\sum_{k=1}^{n}X_k(t)$.

Then, the following inequality holds for any $0<p<1$:

$$\left\|\sup_{t\in T}|S_n(t)|\right\|_{\psi} \leq \breve{B}(p),$$

where $\breve{B}(p)=2\sqrt{C_{\psi}}\inf_{t\in T}\|Y(t)\|_{\psi}+\frac{1}{p(1-p)}\int_0^{\gamma p}\kappa\left(N\left(\sigma_1^{(-1)}(u)\right)\right)du$,

$$\gamma=\sigma_1\left(\sup_{t,s\in T}\rho(t,s)\right)=2\sqrt{C_{\psi}}\sigma\left(\sup_{t,s\in T}\rho(t,s)\right).$$

Proof. It follows from the definition 2.3 that

$$\|S_n(t)-S_n(s)\|_{\psi}^2 = \left\|\frac{1}{\sqrt{n}}\sum_{k=1}^{n}\left(X_k(t)-X_k(s)\right)\right\|_{\psi}^2 \leq C_{\psi}\frac{1}{n}\sum_{k=1}^{n}\|X_k(t)-X_k(s)\|_{\psi}^2 = C_{\psi}\|X(t)-X(s)\|_{\psi}^2$$

and

$$\|X(t)-X(s)\|_{\psi}^{2}=\|Y(t)-Y(s)-(m(t)-m(s))\|_{\psi}^{2}\leq\left(\|Y(t)-Y(s)\|_{\psi}+\|m(t)-m(s)\|_{\psi}\right)^{2}\leq$$

$$\leq\left(\|Y(t)-Y(s)\|_{\psi}+\frac{|m(t)-m(s)|}{\psi(1)}\right)^{2}.$$

Lemma 4.1 implies that

$$|m(t)-m(s)|\leq E\|Y(t)-Y(s)\|_{\psi}\leq\|Y(t)-Y(s)\|_{\psi}\psi(1).$$

Then

$$\|S_n(t)-S_n(s)\|_{\psi}^{2}\leq 4C_{\psi}\|Y(t)-Y(s)\|_{\psi}^{2}.$$

So, we get

$$\sup_{\rho(t,s)\leq h}\|S_n(t)-S_n(s)\|_{\psi}\leq\sigma_1(h),$$

where $\sigma_1(h)=2\sqrt{C_{\psi}}\sigma(h)$.

It is evident that for every $t_0\in T$

$$\|X(t_0)\|_{\psi}^{2}\leq 4C_{\psi}\|Y(t_0)\|_{\psi}^{2}.$$

Therefore, the proof the theorem can be completed according to the theorem 3.1.

***Corollary* 4.19** *If the conditions of the theorem 4.1 are fulfilled for the process* $X=\{X(t),t\in T\}$, *belonging to the space* $F_{\psi}(\Omega)$ *then for any* $\varepsilon>0$ *the following is true:*

$$P\left\{\sup_{t\in T}|S_n(t)|>\varepsilon\right\}\leq\inf_{u\geq 1}\frac{B^u(p)(\psi(u))^u}{\varepsilon^u}.$$

Proof. The proof of the corollary 4.1 follows from the corollary 3.1.

5. Estimates for the distribution of the suprema on R for the stochastic processes from $F_{\psi}(\Omega)$ spaces

***Theorem 5.1*10** Let $X=\{X(t),t\in R\}$ *be a separable stochastic process from the space* $F_{\psi}(\Omega)$, $T_k=[a_k,a_{k+1}]$, $-\infty<a_k<a_{k+1}<+\infty$, $k\in Z$. *For every* T_k *there exists such a continuous, strictly monotonically increasing function* $\sigma_k=\{\sigma_k(h),0\leq h\leq a_{k+1}-a_k\}$, $\sigma_k(0)=0$ *that*

$$\sup_{\substack{|t-s|\leq h\\ t,s\in T_k}}\|X(t)-X(s)\|_{\psi}\leq\sigma_k(h).$$

We assume that the following conditions are true:

1. $\int_0^{\gamma_k}\kappa\left(\frac{a_{k+1}-a_k}{2\sigma_k^{(-1)}(u)}+1\right)du<\infty$, *where* $\gamma_k=\sigma_k\left(\frac{a_{k+1}-a_k}{2}\right)$, $\sigma_k^{(-1)}(u)$ *is an inverse*

function for $\sigma_k(u)$, $\kappa(u)$ *is the majorizing characteristics for the space* $F_\psi(\Omega)$;

2. *The series* $\sum_{k\in Z}\inf_{u\geq 1}\frac{B_k^u(\psi(u))^u}{\varepsilon^u}$ *is convergent. Here* p_k *are some numbers,* $0<p_k<1$,

$$B_k=\inf_{t\in T_k}\|X(t)\|_\psi+\frac{1}{p_k(1-p_k)}\int_0^{\gamma_k p_k}\kappa\left(\frac{a_{k+1}-a_k}{2\sigma_k^{(-1)}(u)}+1\right)du.$$

Then for any $\varepsilon>0$ *the following holds true:*

$$P\left\{\sup_{t\in R}|X(t)|>\varepsilon\right\}\leq\sum_{k\in Z}\inf_{u\geq 1}\frac{B_k^u(\psi(u))^u}{\varepsilon^u}.$$

Proof. The inequality

$$P\left\{\sup_{t\in R}|X(t)|>\varepsilon\right\}\leq\sum_{k\in Z}P\left\{\sup_{t\in T_k}|X(t)|>\varepsilon\right\},$$

and the corollary 3.2 imply that:

$$P\left\{\sup_{t\in R}|X(t)|>\varepsilon\right\}\leq\sum_{k\in Z}\inf_{u\geq 1}\frac{B_k^u(\psi(u))^u}{\varepsilon^u},$$

So, this completes the proof.

Remark 5.1*11* Let us consider the integral $\int_0^{\gamma_k p_k}\kappa\left(\frac{a_{k+1}-a_k}{2\sigma_k^{(-1)}(u)}+1\right)du$. Since for such integral $0<u\leq p_k\sigma_k\left(\frac{a_{k+1}-a_k}{2}\right)\leq\sigma_k\left(\frac{a_{k+1}-a_k}{2}\right)$ or, equivalently,

$$\frac{a_{k+1}-a_k}{2\sigma_k^{(-1)}(u)}\geq\frac{a_{k+1}-a_k}{2\sigma_k^{(-1)}\left(\sigma_k\left(\frac{a_{k+1}-a_k}{2}\right)\right)}=1,$$

then

$$\int_0^{\gamma_k p_k}\kappa\left(\frac{a_{k+1}-a_k}{2\sigma_k^{(-1)}(u)}+1\right)du\leq\int_0^{\gamma_k p_k}\kappa\left(\frac{a_{k+1}-a_k}{\sigma_k^{(-1)}(u)}\right)du.$$

Remark 5.2*12* If $t,s\in T_k$ then $\|X(t)-X(s)\|_\psi\leq\|X(t)\|_\psi+\|X(s)\|_\psi\leq 2\sup_{t\in T_k}\|X(t)\|_\psi$ and therefore it will always be true

$$\sup_{\substack{|t-s|\leq h\\ t,s\in T_k}}\|X(t)-X(s)\|_\psi\leq\min\left(\sigma(h),2\sup_{t\in T_k}\|X(t)\|_\psi\right)=\hat{\sigma}_k(h).$$

The theorem 4.1 will also be valid if in the definition of B_k we put $\gamma_{k1} = \sigma_k\left(\frac{a_{k+1}-a_k}{2}\right)$ or $\gamma_{k2} = 2\sup_{t\in T_k}\|X(t)\|_\psi$ instead of γ_k.

Theorem 5.2 **13***Let* $X = \{X(t), t\in R\}$ *be a separable stochastic process from the space* $F_\psi(\Omega)$ *and* $\psi(u) = u^\alpha$, $\alpha > 0$, $T_k = [a_k, a_{k+1}]$, $-\infty < a_k < a_{k+1} < +\infty$, $k \in Z$. *For every* T_k *there exists such a monotone, strictly increasing function* $\sigma_k = \{\sigma_k(h), 0 \le h \le a_{k+1} - a_k\}$, $\sigma_k(0) = 0$ *that*

$$\sup_{\substack{|t-s|\le h\\ t,s\in T_k}} \|X(t) - X(s)\|_\psi \le \sigma_k(h).$$

Assume that the following conditions hold true:

1. $\int_0^{\gamma_k p_k}\left(\ln\left(1+\frac{a_{k+1}-a_k}{2\sigma_k^{(-1)}(u)}\right)\right)^\alpha du < \infty$, *where* $0 < p_k < 1$, $\gamma_k = \sigma_k\left(\frac{a_{k+1}-a_k}{2}\right)$, $\sigma_k^{(-1)}(u)$ *is an inverse function for* $\sigma_k(u)$;
2. *the series* $\sum_{k\in Z}\exp\left\{-\frac{2\alpha}{e}\left(\frac{B}{B_k}\right)^{1/\alpha}\right\} < \infty$, *where* $B = \sup_{k\in Z} B_k$ *and*

$$B_k = \inf_{t\in T_k}\|X(t)\|_\psi + \frac{1}{p_k(1-p_k)}\left(\frac{e}{\alpha}\right)^\alpha \int_0^{\gamma_k p_k}\left(\ln\left(1+\frac{a_{k+1}-a_k}{2\sigma_k^{(-1)}(u)}\right)\right)^\alpha du.$$

Then for any $\varepsilon > 2^{2\alpha}B$

$$P\left\{\sup_{t\in R}|X(t)| > \varepsilon\right\} \le \exp\left\{-\frac{\alpha}{2e}\left(\frac{\varepsilon}{B}\right)^{1/\alpha}\right\}\cdot\sum_{k\in Z}\exp\left\{-\frac{2\alpha}{e}\left(\frac{B}{B_k}\right)^{1/\alpha}\right\}.$$

Proof. It follows from the theorems 5.1 and 2.2 that

$$P\left\{\sup_{t\in R}|X(t)| > \varepsilon\right\} \le \sum_{k\in Z}\exp\left\{-\frac{\alpha}{e}\left(\frac{\varepsilon}{B_k}\right)^{1/\alpha}\right\}. \tag{3}$$

If $B = \sup_{k\in Z} B_k$, then $\left(\frac{\varepsilon}{B_k}\right)^{1/\alpha} = \left(\frac{\varepsilon}{2^\alpha B}\right)^{1/\alpha}\cdot\left(\frac{2^\alpha B}{B_k}\right)^{1/\alpha}$. Since for $x \ge 2$, $y \ge 2$ it is true that $xy \ge (x+y)$, so if $\varepsilon > 2^{2\alpha}B$ we obtain that

$$\left(\frac{\varepsilon}{B_k}\right)^{1/\alpha} \ge \left(\frac{\varepsilon}{2^\alpha B}\right)^{1/\alpha} + \left(\frac{2^\alpha B}{B_k}\right)^{1/\alpha} = \frac{1}{2}\left(\frac{\varepsilon}{B}\right)^{1/\alpha} + 2\left(\frac{B}{B_k}\right)^{1/\alpha}. \tag{4}$$

The inequalities (3) and (4) imply that for $\varepsilon > 2^{2\alpha} B$

$$P\left\{\sup_{t\in R}|X(t)| > \varepsilon\right\} \le \sum_{k\in Z} \exp\left\{-\frac{\alpha}{e}\left(\frac{1}{2}\left(\frac{\varepsilon}{B}\right)^{1/\alpha} + 2\left(\frac{B}{B_k}\right)^{1/\alpha}\right)\right\} =$$

$$= \exp\left\{-\frac{\alpha}{2e}\left(\frac{\varepsilon}{B}\right)^{1/\alpha}\right\} \cdot \sum_{k\in Z} \exp\left\{-\frac{2\alpha}{e}\left(\frac{B}{B_k}\right)^{1/\alpha}\right\}.$$

This completes the theorem's proof.

Example 5.1 **14**Let $X(t)$ be a stochastic process from the space $F_\psi(\Omega)$ for which $X(-t) = X(t)$ and

$$\sigma_k(h) = \frac{C}{\left(\ln\left(\frac{a_{k+1} - a_k}{2h} + 1\right)\right)^{\alpha/\mu}},$$

where $\mu < 1$. Then $B_k = B_{-k}$ and the corollary 3.3 implies that for $k > 0$

$$B_k = \inf_{t\in T_k} \|X(t)\|_\psi + C_k \cdot f(\mu),$$

where $f(\mu) = \left(\frac{e}{\alpha}\ln\frac{3}{2}\right)^{(\mu-1)/\mu} \frac{(1+\mu)^{\mu+1}}{\mu^\mu(1-\mu)}$. We denote $Z_k = \inf_{t\in T_k} \|X(t)\|_\psi$ then

$$B_k = Z_k + C_k \cdot f(\mu) \le \max(Z_k, C_k)(1 + f(\mu)) = \widetilde{B}_k.$$

So, the theorem 5.2 holds true if the following series converges

$$\sum_{k=0}^{\infty} \exp\left\{-\frac{2\alpha}{e}\left(\frac{\widetilde{B}}{\widetilde{B}_k}\right)^{1/\alpha}\right\}, \tag{5}$$

here $\widetilde{B} = \sup_{k\in Z} \widetilde{B}_k$. It is evident that this series is convergent if

$$\frac{\widetilde{B}}{\widetilde{B}_k} \ge C(\ln k)^{\alpha+\varepsilon}, \tag{6}$$

where $C > 0$ is some constant. Since

$$\exp\left\{-\frac{2\alpha}{e}\left(\frac{\widetilde{B}}{\widetilde{B}_k}\right)^{1/\alpha}\right\} \le \exp\left\{-\frac{2\alpha}{e} C^{\frac{1}{\alpha}} (\ln k)^{1+\frac{\varepsilon}{\alpha}}\right\} = k^{-\left(\frac{2\alpha}{e} C^{\frac{1}{\alpha}} (\ln k)^{\frac{\varepsilon}{\alpha}}\right)}$$

and for sufficiently large k

$$\left(\frac{2\alpha}{e}C^{\frac{1}{\alpha}}(\ln k)^{\frac{\varepsilon}{\alpha}}\right) > 1,$$

then the series (5) is convergent.

Remark15 5.3 The condition (6) can be written as: there exists $D > 0$ that $\tilde{B}_k \le D \cdot \frac{1}{(\ln k)^{\alpha+\varepsilon}}$, as $\varepsilon > 0$.

Example 5.2 *16* Let $X(t)$ be such a stochastic process from the space $F_\psi(\Omega)$ that $X(-t) = X(t)$. If $\sigma_k(h) = \overline{C}_k |h|^\delta$, $0 < \delta \le 1$, then

$$B_k = \inf_{t \in T_k} \|X(t)\|_\psi + \tilde{A}\overline{C}_k^{-\frac{\alpha\tau}{\delta}} = \breve{B}_k,$$

where $\tilde{A} = \dfrac{\left(\frac{e}{\alpha\tau}\right)^\alpha \left(\frac{d-c}{2}\right)^{\alpha\tau} (\gamma p)^{1-\frac{\alpha\tau}{\delta}}}{p(1-p)\left(1-\frac{\alpha\tau}{\delta}\right)}$, $0 \le \tau \le 1$. We denote $\breve{B} = \sup_{k \in Z} \breve{B}_k$. Then the conditions of the theorem 5.2 are fulfilled if

$$\breve{B}_k \le D\frac{1}{(\ln k)^{\alpha+\varepsilon}},$$

where $D > 0$, $\varepsilon > 0$.

Theorem 5.3 **17** *Let $X = \{X(t), t \in R\}$ be a separable stochastic process from the space $F_\psi(\Omega)$ and $\psi(u) = e^{au^\beta}$, $a > 0$, $\beta > 0$, $T_k = [a_k, a_{k+1}]$, $-\infty < a_k < a_{k+1} < +\infty$, $k \in Z$. For every T_k there exists such a continuous, monotonously increasing function $\sigma_k = \{\sigma_k(h), 0 \le h \le a_{k+1} - a_k\}$, $\sigma_k(0) = 0$ that*

$$\sup_{\substack{|t-s| \le h \\ t,s \in T_k}} \|X(t) - X(s)\|_\psi \le \sigma_k(h).$$

If the following conditions are fulfilled:

1. $\displaystyle\int_0^{\gamma_k p_k} \frac{1}{e^a} \exp\left\{S(a,\beta)\ln\left(\frac{a_{k+1}-a_k}{2\sigma_k^{(-1)}(u)}+1\right)\right\} du < \infty$, *where* $S(a,\beta) = (\beta a)^{\frac{1}{\beta+1}}(\beta^{-1}+1)$, $0 < p_k < 1$, $\gamma_k = \sigma_k\left(\frac{a_{k+1}-a_k}{2}\right)$;

2. the series $\sum_{k\in Z}\exp\left\{-R_\beta\left(\ln\frac{B}{B_k}\right)^{\frac{\beta+1}{\beta}}\right\}$ *converges with* $R_\beta=\frac{\beta}{a^{\frac{1}{\beta}}(\beta+1)^{\frac{\beta+1}{\beta}}}$,

$$B_k=\inf_{t\in T_k}\|X(t)\|_\psi+\frac{1}{p_k(1-p_k)}\frac{1}{e^a}\int_0^{\gamma_k p_k}\exp\left\{S(a,\beta)\ln\left(\frac{a_{k+1}-a_k}{2\sigma_k^{(-1)}(u)}+1\right)\right\}du,$$

$B=\sup_{k\in Z}B_k$,

then for any $\varepsilon>B$ *the following inequality holds true:*

$$P\left\{\sup_{t\in R}|X(t)|>\varepsilon\right\}\le\exp\left\{-R_\beta\left(\ln\frac{\varepsilon}{B}\right)^{\frac{\beta+1}{\beta}}\right\}\cdot\sum_{k\in Z}\exp\left\{-R_\beta\left(\ln\frac{B}{B_k}\right)^{\frac{\beta+1}{\beta}}\right\}.$$

Proof. The theorem's statement follows from the theorems 5.1, 2.3 and 2.5. Indeed:

$$P\left\{\sup_{t\in R}|X(t)|>\varepsilon\right\}\le\sum_{k\in Z}\exp\left\{-R_\beta\left(\ln\frac{\varepsilon}{B_k}\right)^{\frac{\beta+1}{\beta}}\right\},$$

and since $\frac{\beta+1}{\beta}>1$ then

$$\left(\ln\frac{\varepsilon}{B_k}\right)^{\frac{\beta+1}{\beta}}=\left(\ln\left(\frac{\varepsilon}{B}\cdot\frac{B}{B_k}\right)\right)^{\frac{\beta+1}{\beta}}\ge\left(\ln\frac{\varepsilon}{B}\right)^{\frac{\beta+1}{\beta}}+\left(\ln\frac{B}{B_k}\right)^{\frac{\beta+1}{\beta}}.$$

The last inequality impliles that:

$$P\left\{\sup_{t\in R}|X(t)|>\varepsilon\right\}\le\exp\left\{-R_\beta\left(\ln\frac{\varepsilon}{B}\right)^{\frac{\beta+1}{\beta}}\right\}\cdot\sum_{k\in Z}\exp\left\{-R_\beta\left(\ln\frac{B}{B_k}\right)^{\frac{\beta+1}{\beta}}\right\},$$

This completes the proof.

Theorem 5.4 **18** *Let* $X=\{X(t),t\in R\}$ *be a separable stochastic process from the space* $F_\psi(\Omega)$, $T_k=[a_k,a_{k+1}]$, $-\infty<a_k<a_{k+1}<+\infty$, $k\in Z$. *Assume that for some* $0<\mu<1$ *the following condition is fulfilled:*

$$\sup_{\substack{|t-s|\le h\\ t,s\in T_k}}\|X(t)-X(s)\|_\psi\le\frac{C_k}{\left(\kappa\left(\frac{T_k}{2h}+1\right)\right)^{1/\mu}},$$

where $h>0$. *Then* $\sup_{t\in T_k}|X(t)|\in F_\psi(\Omega)$ *with probability one and the following inequality holds*

$$\left\|\sup_{t\in T_k}|X(t)|\right\|_\psi \le \inf_{t\in T_k}\|X(t)\|_\psi + C_k\left(\kappa\left(\frac{3}{2}\right)\right)^{(\mu-1)/\mu}\frac{(1+\mu)^{\mu+1}}{\mu^\mu(1-\mu)} = \widetilde{B}_k.$$

Moreover, for any $\varepsilon > 0$ the following is also true:

$$P\left\{\sup_{t\in R}|X(t)| > \varepsilon\right\} \le \sum_{k\in Z}\inf_{u\ge1}\frac{\widetilde{B}_k^u(\psi(u))^u}{\varepsilon^u}.$$

Proof. The proof of the theorem follows from the corollary 3.3 and theorem 5.1.

***Theorem 5.5* 19** *Let $X = \{X(t), t\in R\}$ be a separable stochastic process from the space $F_\psi(\Omega)$, $T_k = [a_k, a_{k+1}]$, $-\infty < a_k < a_{k+1} < +\infty$, $k\in Z$. For every T_k there exists such a continuous, strictly monotonously increasing function $\sigma_k(h)$, $0 \le h \le (a_{k+1}-a_k)$, $\sigma_k(0)=0$ that*

$$\sup_{\substack{|t-s|\le h\\ t,s\in T_k}}\|X(t)-X(s)\|_\psi \le \sigma_k(h).$$

There also exists some continuous function $c = \{c(t), t\in R\}$ that $c(t) > 1$, $r_k = \inf_{t\in T_k} c(t)$.

If the following conditions are fulfilled:

1. $\int_0^{\gamma_k}\kappa\left(\frac{a_{k+1}-a_k}{2\sigma_k^{(-1)}(u)}+1\right)du < \infty$, *where* $\gamma_k = \sigma_k\left(\frac{a_{k+1}-a_k}{2}\right)$;
2. *the series* $\sum_{k\in Z}\inf_{u\ge1}\frac{B_k^u(\psi(u))^u}{(\varepsilon r_k)^u}$ *converges ; here*

$B_k = \inf_{t\in T_k}\|X(t)\|_\psi + \frac{1}{p_k(1-p_k)}\int_0^{\gamma_k p_k}\kappa\left(\frac{a_{k+1}-a_k}{2\sigma_k^{(-1)}(u)}+1\right)du$, p_k *are some numbers* $0 < p_k < 1$, $\kappa(n)$ *is the majorizing characteristics of the space* $F_\psi(\Omega)$;

then for any $\varepsilon > 0$ the following estimate is true:

$$P\left\{\sup_{t\in R}\frac{|X(t)|}{c(t)} > \varepsilon\right\} \le \sum_{k\in Z}\inf_{u\ge1}\frac{B_k^u(\psi(u))^u}{(\varepsilon r_k)^u}.$$

Proof. It is evident that

$$P\left\{\sup_{t\in R}\frac{|X(t)|}{c(t)} > \varepsilon\right\} \le \sum_{k\in Z}P\left\{\sup_{t\in T_k}\frac{|X(t)|}{c(t)} > \varepsilon\right\} \le \sum_{k\in Z}P\left\{\sup_{t\in T_k}|X(t)| > \varepsilon r_k\right\}.$$

From the corollary 3.2 we get that

$$P\left\{\sup_{t\in T_k}|X(t)| > \varepsilon r_k\right\} \le \inf_{u\ge1}\frac{B_k^u(\psi(u))^u}{(\varepsilon r_k)^u}.$$

So,

$$P\left\{\sup_{t\in R}\frac{|X(t)|}{c(t)}>\varepsilon\right\}\le\sum_{k\in Z}\inf_{u\ge1}\frac{B_k^u(\psi(u))^u}{(\varepsilon r_k)^u}.$$

Remark 205.4 The theorem 5.5 implies that there exists such a random variable $\xi>0$ that $\frac{|X(t)|}{c(t)}<\xi$ with probability one.

Theorem 5.621 *Let* $X=\{X(t),t\in R\}$ *be a separable stochastic process from the space* $F_\psi(\Omega)$, *where* $\psi(u)=u^\alpha$, $\alpha>0$, $T_k=[a_k,a_{k+1}]$, $-\infty<a_k<a_{k+1}<+\infty$, $k\in Z$. *There exists such a continuous, strictly monotonously incrasing function* $\sigma(h)$, $\sigma(0)=0$, $\sigma(+\infty)=\gamma<\infty$ *that*

$$\sup_{\substack{|t-s|\le h\\ t,s\in T_k}}\|X(t)-X(s)\|_\psi\le\sigma(h).$$

Assume that the following conditions are fulfilled:

1. $\int_0^\gamma\left(\ln\left(1+\frac{1}{2\sigma_k^{(-1)}(u)}\right)\right)^\alpha du<\infty$;
2. *there exists some continuous function* $c=\{c(t),t\in R\}$ *that* $c(t)>1, t\in R, r_k=\inf_{t\in T_k}c(t)$;
3. *the series* $\sum_{k\in Z}\exp\left\{-\frac{2\alpha}{e}\left(\frac{r_k\tilde{B}}{B_k}\right)^{1/\alpha}\right\}$ *converges with* $\tilde{B}=\sup_{k\in Z}\frac{B_k}{r_k}$,

$$B_k=\inf_{t\in T_k}\|X(t)\|_\psi+\frac{1}{p(1-p)}\left(\frac{e}{\alpha}\right)^\alpha\int_0^{\gamma p}\left(\ln\left(1+\frac{a_{k+1}-a_k}{2\sigma^{(-1)}(u)}\right)\right)^\alpha du,$$

then for any $\varepsilon>2^\alpha\tilde{B}$ *the following inequality holds true:*

$$P\left\{\sup_{t\in R}|X(t)|>\varepsilon\right\}\le\exp\left\{-\frac{\alpha}{2e}\left(\frac{\varepsilon}{\tilde{B}}\right)^{1/\alpha}\right\}\cdot\sum_{k\in Z}\exp\left\{-\frac{2\alpha}{e}\left(\frac{r_k\tilde{B}}{B_k}\right)^{1/\alpha}\right\}.$$

Proof. The proof of the theorem follows from the theorem 5.5 and can be performed in a similar way as the proof of the theorem 5.2.

Corollary 5.122 *Let* $X(t)$ *be such a stochastic process from the space* $F_\psi(\Omega)$ *that* $X(-t)=X(t)$. *Then the condition 3 of the theorem 5.6 holds if there exists such a constan* $D>0$ *that*

$$\frac{B_k}{r_k}\le D\cdot\frac{1}{(\ln k)^{\alpha+\varepsilon}},$$

where $\varepsilon>0$.

Proof. The proof of the the corollary follows from the example 5.1.

Corollary 5.2 **23** *If* $\sup_{t\in R}\|X(t)\|_{\psi} < \infty$ *, then the condition 3 of the theorem 5.6 is fulfilled if*

$$\frac{\left(\ln\left(1+a_{k+1}-a_k\right)\right)^{\alpha}}{r_k} \le \frac{D}{(\ln k)^{\alpha+\varepsilon}}, \qquad (7)$$

where D is some constant, $\varepsilon > 0$.

Proof. Let's put $p = \frac{1}{2}$. It is evident that

$$\int_0^{\frac{\gamma}{2}}\left(\ln\left(1+\frac{a_{k+1}-a_k}{2\sigma^{(-1)}(u)}\right)\right)^{\alpha} du \le \int_0^{\frac{\gamma}{2}}\left(\ln\left(1+\frac{1}{2\sigma^{(-1)}(u)}\right)+\ln\left(1+a_{k+1}-a_k\right)\right)^{\alpha} du \le$$

$$\le C_{\alpha}\int_0^{\frac{\gamma}{2}}\left(\ln\left(1+\frac{1}{2\sigma^{(-1)}(u)}\right)\right)^{\alpha} du + C_{\alpha}\frac{\gamma}{2}\left(\ln\left(1+a_{k+1}-a_k\right)\right)^{\alpha},$$

where $C_{\alpha} = \begin{cases} 2^{\alpha-1} & ,\alpha > 1 \\ 1 & ,\alpha < 1. \end{cases}$ Therefore,

$$B_k \le S_1 + S_2\left(\ln\left(1+a_{k+1}-a_k\right)\right)^{\alpha},$$

where S_1 and S_2 are some constants. So, the inequality (7) is true and the corollary 5.2 holds.

Corollary 5.3 **24** *The condition 3 of the theorem 5.6 holds true if*

$$c(t) = (\ln t)^{\alpha}(\ln\ln t)^{\alpha+\varepsilon},$$

where $\varepsilon > 0$, $t \ge e^2$.

Proof. If we put $a_k = e^k$, $k \ge 1$ then

$$\left(\ln\left(1+a_{k+1}-a_k\right)\right)^{\alpha} = \left(\ln\left(1+e^k(e-1)\right)\right)^{\alpha} \le k^{\alpha}\left(\ln\left(1+e(e-1)\right)\right)^{\alpha}.$$

Therefore

$$\frac{\left(\ln\left(1+a_{k+1}-a_k\right)\right)^{\alpha}}{c\left(e^k\right)} \le \frac{k^{\alpha}\left(\ln\left(1+e(e-1)\right)\right)^{\alpha}}{k^{\alpha}(\ln k)^{\alpha+\varepsilon}} \le \frac{\left(\ln\left(1+e^2-e\right)\right)^{\alpha}}{(\ln k)^{\alpha+\varepsilon}}.$$

Example 5.3 Let us consider the process $X = \{X(t), t \in R\}$, where $X(t) = \sum_{k=1}^{\infty}\xi_k L_k(t)$ and the random variables ξ_k belong to the space $F_{\psi}(\Omega)$ with $\psi(u) = u^{\alpha}$, $\alpha \ge \frac{1}{2}$. For this space the condition **H** is fulfilled. Let the functions $L_k(t)$ satisfy the Lipshitz condition:

$$\left|L_k(t) - L_k(s)\right| \le C_k\left|t-s\right|^{\gamma},$$

where γ is some constant, $0 < \gamma \le 1$ and $C_k > 0$. Then

$$\|X(t)-X(s)\|_{\psi} \le \sum_{k=1}^{\infty}\|\xi_k\|_{\psi}|L_k(t)-L_k(s)| \le \left(\sum_{k=1}^{\infty}\|\xi_k\|_{\psi} C_k\right)|t-s|^{\gamma}.$$

So, if the series $\sum_{k=1}^{\infty}\|\xi_k\|C_k$ converges then

$$\sup_{\substack{|t-s|\le h \\ t,s\in[c,d]}} \|X(t)-X(s)\|_{\psi} \le \breve{C}h^{\gamma},$$

with $\breve{C} = \sum_{k=1}^{\infty}\|\xi_k\|_{\psi} C_k$.

Since $\sigma(h) = \breve{C}h^{\gamma}$, so it is evident that for this process the condition 1 of the theorem 5.6. is fulfilled. As far as

$$\|X(t)\|_{\psi}^2 \le C_{\psi}\sum_{k=1}^{\infty}\|\xi_k\|_{\psi}^2 L_k^2(t),$$

where C_{ψ} is the constant from the definition 2.3, so if $\sup_{t\in R}\sum_{k=1}^{\infty}\|\xi_k\|_{\psi}^2 L_k^2(t) = \hat{C} < +\infty$ the statement of the theorem 5.6 holds true for the process $X(t)$ with function $c(t) = (\ln t)^{\alpha}(\ln\ln t)^{\alpha+\varepsilon}$, $\varepsilon > 0$.

Theorem 5.7 **25***Let* $Y = \{Y(t), t\in R\}$ *be a stochastic process from the space* $F_{\psi}(\Omega)$ *fulfilling the condition* $\mathbf{H}$ *with the constant* C_{ψ}, $T_k = [a_k, a_{k+1}]$, $-\infty < a_k < a_{k+1} < +\infty$, $k\in Z$. *Assume for every* T_k *there exists such a continuous, strictly monotonously increasing function* $\sigma_k = \{\sigma_k(h), 0\le h\le a_{k+1}-a_k\}$, $\sigma_k(0)=0$ *that*

$$\sup_{\substack{|t-s|\le h \\ t,s\in T_k}}\|Y(t)-Y(s)\|_{\psi} \le \sigma_k(h).$$

Let $X(t) = Y(t) - m(t)$, *where* $m(t) = EX(t)$ *and* $X_k(t)$ *are the independent copies of* $X(t)$, $S_n(t) = \frac{1}{\sqrt{n}}\sum_{k=1}^{n}X_k(t)$. *Suppose, the following conditions hold true:*

1. $\int_0^{\gamma_k}\kappa\left(\frac{a_{k+1}-a_k}{2\sigma_{1k}^{(-1)}(u)}+1\right)du < \infty$, *where* $\gamma_k = \sigma_k\left(\frac{a_{k+1}-a_k}{2}\right)$, $\sigma_{1k}(u) = 2\sqrt{C_{\psi}}\sigma_k(u)$;
2. *the series* $\sum_{k\in Z}\inf_{u\ge 1}\frac{\widetilde{B}_k^u(\psi(u))^u}{\varepsilon^u}$ *is convergent with*

$$\widetilde{B}_k = 2\sqrt{C_{\psi}}\inf_{t\in T_k}\|Y(t)\|_{\psi} + \frac{1}{p_k(1-p_k)}\int_0^{\gamma_k p_k}\kappa\left(\frac{a_{k+1}-a_k}{2\sigma_{1k}^{(-1)}(u)}+1\right)du\ ,$$ *p_k being such*

numbers that $0 < p_k < 1$, $\kappa(u)$ *being a majorizing characteristis of the space* $F_\psi(\Omega)$.
Then for any $\varepsilon > 0$ *we'll get:*

$$P\left\{\sup_{t\in R}|S_n(t)| > \varepsilon\right\} \le \sum_{k\in Z}\inf_{u\ge 1}\frac{\widetilde{B}_k^u(\psi(u))^u}{\varepsilon^u}.$$

Proof. The statement of the theorem follows from the theorems 4.1, 5.1 and the corollary 4.1.

6. Conclusions

So, in the paper we have estimated the probabilities of large deviations for the sums of independent stochastic processes from the spaces $F_\psi(\Omega)$. It has been found the estimates for the distribution of suprema on R for the stochastic processes from the $F_\psi(\Omega)$ spaces.

References

Albin, J. M. P. (1998). On extremal theory for self-similar processes. Ann. Probab., 26, 723-793. http://dx.doi.org/10.1214/aop/1022855649

Belyaev, Y. K. (1961). Continuity and Holder's conditions for sample functions of stationary Gaussian processes. Proc. Fourth Berkeley Symp. on Math. and Probability, 2, 23-33.

Belyaev, Yu. K. (1968). On the number of boundary out-crossings of a region by a vector stochastic process. Theory of Probability and its Applications, 13(2), 320-324. http://dx.doi.org/10.1137/1113036

Berman, S. M. (1973). Excursions of stationary Gaussian processes above high moving barriers. The Annals of Probability, 1(3), 133-184. http://dx.doi.org/10.1214/aop/1176996932

Bondarenko, I. V. & Ivanov, A. V. (1992). On sample properties of random fields with stable increments. Theory Probab. Math. Stat., 47, 11-15.

Borell, C. (1978). Tail probabilities in Gaussian space. Vector space measures. Appl. I. Lect. Notes Math., 644, 73-82. http://dx.doi.org/10.1007/BFb0066841

Buldygin, V. V. (1977). Sub-Gaussian processes and the convergence of random series in function spaces. Ukrain. Math. J., 29(4), 337-345. http://dx.doi.org/10.1007/BF01085798

Buldygin, V. V. & Kozachenko, Yu. V. (1974). On local properties of realizations of some stochastic processes and fields. Theory Probab. Math. Stat., 10, 37-45.

Buldygin, V. V. & Kozachenko, Yu. V. (1980). On sub-Gaussian random variables, Ukrain. Math. J., 32(6), 483-489. http://dx.doi.org/10.1007/BF01087176

Buldygin, V. V. & Kozachenko, Yu. V. (1993). Estimates for the distribution of the supremum of a class of random processes. Ukrain. Math. J., 45(5), 647-661. http://dx.doi.org/10.1007/BF01058203

Buldygin, V. V. & Kozachenko, Yu. V. (2000) Metric characterization of random variables and random processes. Providence, RI: AMS, American Mathematical Society. xii, 257.

Cramer, H. & Leadbetter. M. R. (1967). Stationary and related stochastic processes. Sample function properties and their applications. New York–London–Sydney: Wiley, 348.

Delporte, J. (1964) Fonctions alétoires presque sûrement continues sur un intervalle fermé. Annales de l'I H. P., 1(2), 111-215.

Dmitrovskij, V. A. (1981). On the distribution of the maximum and local properties of realizations of pre-Gaussian fields. Theory Probab. Math. Stat., 25, 154-164.

Dudley, R. M. (1965). Gaussian processes on several parameters. Ann. Math. Statist., 36(3), 771-788. http://dx.doi.org/10.1214/aoms/1177700052

Dudley, R. M. (1973). Sample functions of the Gaussian processes. The Annals of Probability, 1(1), 3-68. http://dx.doi.org/10.1214/aop/1176997026

Fukuda, R. (1990). Exponential integrability of sub-Gaussian vectors. Probab. Theory Related Fields, 85(4), 505-521.
http://dx.doi.org/10.1007/BF01203168

Giuliani, A. R., Kozachenko, Yu. V. & Nikitina, T. (2003). Spaces of -subgaussian random variables. Rediconti Accademia Nazionale dell Scienze detta dei XL, Memorie di Matimatica e Applicazioni, 121, 95-124.

Giuliano, A. R., Kozachenko, Yu. V. & Tegza, A. M. (2002). Inequalities for the norms of sub-Gaussian vectors and the accuracy of the modelling of random processes. Theory Probab. Math. Stat., 66, 58-66.

Iadrenko, M. (1983). Spectral theory of random fields. Springer, 259.

Jain, N. C. & Marcus, M. B. (1978). Continuity of sub-Gaussian processes. Adv. Probab., 4 81-196.

Kahane, J. P. (1960). Propriétés locales des fonctions à series de Fouries aléatories. Studia Math., 19(1) 1-25.

Kahane, J. P. (1968). Some random series of functions. Heath Mathematical Monographs, 184.

Kozachenko, Yu. V. (1968). Sufficient conditions for the continuity with probability one of the sub-Gaussian stochastic processes. Reports of NASU, 2, 113-115. (In Ukrainian)

Kozachenko, Yu. V. & Mlavets, Yu. Yu. (2012). Banach spaces of random variables . Theory Probab. Math. Stat., 86, 92-107.

Kozachenko, Yu. V. & Ostrovskij, E. I. (1985). Banach spaces of random variables of sub-Gaussian type. Theory Probab. Math. Stat., 32, 42-53.

Kozachenko, Y. V., Vasylyk, O. & Yamnenko, R. (2003). Upper estimate of overrunning by random process the level specified by continuous function. Random Oper. and Stochastic Equations, 11(1), 1-20. http://dx.doi.org/10.1515/156939703322003953

Kozachenko, Yu. V. & Yadrenko, M. J. (1976). Local properties of sample functions of random fields. Theory Probab. Math. Stat., 14, 53-66.

Landau, H. J. & Shepp, L. A. (1970). On the supremum of Gaussian processes. Sankhya, 32(4), 369-378.

Ledoux, M. (1990). A note on large deviations for Wiener chaos. Lecture Notes in Mathematics, 1426, 1-14. http://dx.doi.org/10.1007/BFb0083753

Ledoux, M. & Talagrand, M. (1991). Probability in Banach space. Berlin-New York: Springer-Verlag, 480. http://dx.doi.org/10.1007/978-3-642-20212-4

Lifshits M. A. (1995). Gaussian Random Functions. Springer, 333.

Lindgren, G. (1971). Extreme values of stationary normal processes. Z. Wahrscheinlichkeitstheor. Verw. Geb., 17, 39-47.
http://dx.doi.org/10.1007/BF00538473

Marcus, M. B. & Pisier, G. (1981). Random Fourier series with applications to harmonic analysis. Prinston: Prinston University Press, 152.

Ostrovskii, E. I. (1990). Exponential estimates of the distribution of the maximum of a non-Gaussian random field. Theory of Probability and its Applications, 35(3), 487-499. http://dx.doi.org/10.1137/1135068

Pashko, A. A. (1998). Uniform convergence of sub-Gaussian integrals. Theory of Probability and its

Applications, 43(4), 650-655.
http://dx.doi.org/10.1137/S0040585X97977239

Piterbarg, V. I. (1982). Large deviations of random processes close to Gaussian processes. Theory of Probability and its Applications, 27(3), 504-524.
http://dx.doi.org/10.1137/1127059

Piterbarg, V. I. (1996). Asymptotic methods in the theory of Gaussian processes and fields. Providence: AMS, 205.

Skorokhod, A. V. (1970). A remark on Gaussian measures in a Banach space. Theory of Probability and its Applications, 15(3), 508.
http://dx.doi.org/10.1137/1115052

Skorokhod, A. V. (1973). A theorem on the continuity of a random function on a compact set in Hilbert space. Theory of Probability and its Applications, 18(4), 809-811.

Slutsky, E. (1937). Alcuno proposizioni sulla teoria delle funzioni aleatorie. Giorn. Inst. Italiano degli Attuari, 8, 193–199.

Talagrand, M. (1987). Regularity of Gaussian processes. Acta Math., 159, 99-149.
http://dx.doi.org/10.1007/BF02392556

Vasylyk, O. I., Kozachenko, Yu. V. & Yamnenko, R. Ye. (2008). -sub-Gaussian stochastic processes. Kyiv: P "Kyiv University ", 231. (In Ukrainian)